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Contrasting Discrete and Continuous Time Methods for Bayesian System Identification

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Background

Often we want to identify models governed by stochastic differential equations, but usually we use discrete time models for learning.

$$x_{t+\delta} = f_{DT}(x_t) + L_{DT}\kappa_t \quad \text{or} \quad dx_t = f_{CT}(x_t)\,dt + L_{CT}\,d\beta_t$$

$$y_i = g(x_{t_i}) + \rho_i$$

Each $\kappa_t \sim \mathcal{N}(0, I)$ independently, each $\rho_i \sim \mathcal{N}(0, R)$ independently. β_t is standard D-dimensional Brownian motion.

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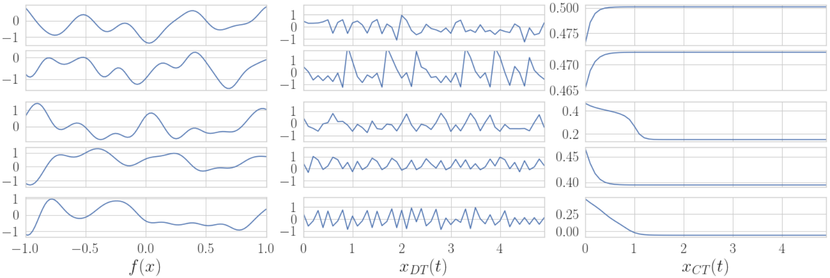
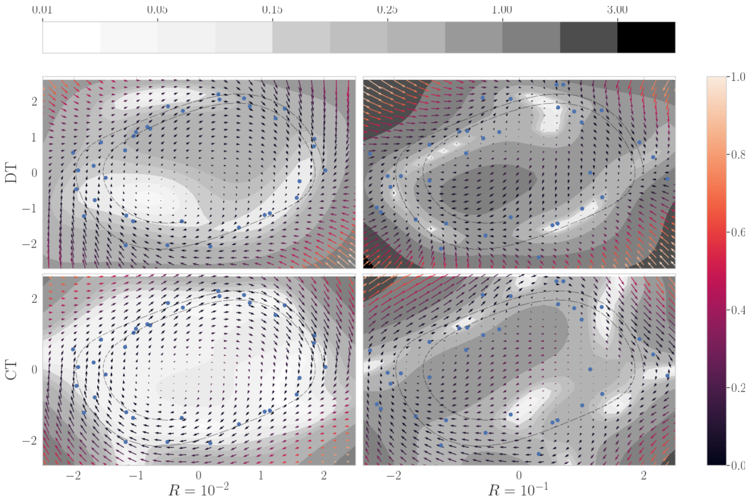
Bayesian modelling: $f \sim \mathcal{GP}(m, k)$

Contrasting priors



The prior samples are qualitatively different unless f_{DT} is a diffeomorphism.

Contrasting posteriors



Conclusions

- Discrete and continuous time priors have important differences
- These impact the quality of posteriors in challenging learning scenarios
- But CT models are expensive and challenging–a good way of constructing diffeomorphic DT priors would be appealing.